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Dynamics of coupled light waves and electron-acoustic waves

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The nonlinear interaction between coherent light waves and electron-acoustic waves in a two-electron plasma is considered. The interaction is governed by a pair of equations comprising a Schrödinger-like equation for the light wave envelope and a driven (by the light pressure) electron-acoustic wave equation. The newly derived nonlinear equations are used to study the formation and dynamics of envelope light wave solitons and light wave collapse. The implications of our investigation to space and laser-produced plasmas are pointed out.

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I. INTRODUCTION

About four decades ago Karpman [1,2] investigated the nonlinear interaction between light and ion-acoustic waves in an unmagnetized plasma. The Karpman equations admit both envelope soliton and shocklike solutions [1–3]. On the other hand, light waves can also nonlinearly interact with electron plasma waves (the Langmuir waves) through the light pressure. The resulting interaction is then governed by a nonlinear Schrödinger equation for the light and a Langmuir wave equation containing light waves as a driver. Studies of nonlinearly coupled light and ion-acoustic/Langmuir waves are of practical interest with regard to the understanding of the complex cooperative phenomena in laser-plasma interactions [4–6], in plasma based charged particle accelerators [7], and in space plasmas [8–10].

However, there is conclusive evidence [5,9–13] that laboratory and space plasmas can contain two distinct groups of electrons. In such plasmas there are electron-acoustic waves (EAWs) [14–16] in which the restoring force comes from the pressure of the inertialess hot electrons, and the mass of the cold electrons provides the inertia. The wave frequency ω and the wave number k are related by

$$\omega=\frac{kC_e}{(1+k^2\lambda_{Dh}^2)^{1/2}},$$

where $C_e=\lambda_{Dh}\omega_{pc}$ is the electron-acoustic speed, $\lambda_{Dh}=(T_h/4\pi n_{h0}e^2)^{1/2}$ is the electron Debye length involving the temperature T_h of the hot-electron component with the equilibrium density n_{h0}, $\omega_{pc}=(4\pi n_{c0}e^2/m)^{1/2}$ is the plasma frequency of the cold-electron component with the equilibrium density n_{c0}, e is the magnitude of the electron charge, and m is the electron mass. The phase velocity of the EAWs is between the thermal speeds of the hot- and cold-electron species, and the wave frequency is larger than the ion plasma frequency. Therefore, the ions do not participate in the EA wave dynamics and they form only a neutralizing background. The ratio between the cold electron and ion number densities is supposed to be much larger than the square root of the electron to ion mass ratio.

In this paper, we consider the nonlinear interaction between light and electron-acoustic waves in a two-electron plasma with a fixed ion background. We first derive the light wave equation in the presence of the density perturbations of the EAWs. The equations for the latter, including the light wave pressure, are obtained by using the continuity and momentum equations for the cold electrons as well as a modified Boltzmann electron number density for the hot electrons, and Poisson's equation. The driven (by the light pressure) electron-acoustic wave responses are significantly different from those involving Langmuir waves. The newly derived equations for nonlinearly coupled light and EAWs can be used to study the formation of light envelope solitons, collapse of light, and plasma density holes trapping light.

II. DERIVATION OF THE NONLINEAR EQUATIONS

We consider a uniform plasma containing large amplitude light waves that are nonlinearly interacting with EAWs. The plasma constituents are singly charged ions, as well as hot and cold electrons. The dynamics of the light waves is governed by Maxwell's equation

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \tag{1}$$

in which the wave magnetic and electric fields are $\mathbf{B}=\nabla\times\mathbf{A}$ and $\mathbf{E}=-c^{-1}\partial\mathbf{A}/\partial t$, where $\mathbf{A}$ is the vector potential and c is the speed of light in vacuum. Furthermore, the electron current density is

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$$\mathbf{J}=-e(n_h+n_c)\mathbf{v}, \tag{2}$$

where n_h and n_c are the number densities of the hot- and cold-electron components, and $\mathbf{v}$ is the electron quiver velocity in the light fields. It is given by

$$\frac{\partial \mathbf{v}}{\partial t}=-\frac{e}{m}\mathbf{E}\equiv\frac{e}{mc}\frac{\partial \mathbf{A}}{\partial t}, \tag{3}$$

which yields

$$\mathbf{v}=\frac{e}{mc}\mathbf{A}. \tag{4}$$

We decompose $n_h=n_{h0}+n_{hs}$ and $n_c=n_{c0}+n_{cs}$, where $n_{h0}+n_{c0}=n_0$ and $n_{hs}(n_{cs})$ is the density perturbation of the hot- (cold-) electron component involved in the EAWs, and combine Eqs. (1)–(4) to obtain

$$\frac{\partial^2 \mathbf{A}}{\partial t^2}-c^2\nabla^2\mathbf{A}+\omega_p^2\mathbf{A}+\omega_{ph}^2 N\mathbf{A}=0, \tag{5}$$

where we have introduced a Coulomb gauge $\boldsymbol{\nabla}\cdot\mathbf{A}=0$ and denoted $\omega_p=(\omega_{ph}^2+\omega_{pc}^2)^{1/2}$, $\omega_{ph}=(4\pi n_{h0}e^2/m)^{1/2}$, and $N=(n_{hs}+n_{cs})/n_{h0}\equiv N_h+N_c\delta$, with $N_h=n_{hs}/n_{h0}, N_c=n_{cs}/n_{c0}$ and $\delta=n_{c0}/n_{h0}$.

Assuming that the nonlinear interactions between the light and electron-acoustic perturbations produce a slowly varying light envelope, we represent

$$\mathbf{A}=\psi(\tau,z)(\hat{\mathbf{x}}+i\hat{\mathbf{y}})\exp(-i\omega_0 t+ik_0 z), \tag{6}$$

where $\omega_0=(k_0^2c^2+\omega_p^2)^{1/2}$ is the light wave frequency, $\psi(\tau,z)$ is a slowly varying envelope, and $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x and y axes, respectively. Inserting Eq. (6) into Eq. (5) and invoking the WKB approximation, viz. $|\partial\psi/\partial\tau|\ll\omega_0\psi$, we obtain the modulated light wave equation,

$$2i\omega_0\left(\frac{\partial}{\partial \tau}+v_g\frac{\partial}{\partial z}\right)\psi+c^2\frac{\partial^2\psi}{\partial z^2}-\omega_{ph}^2 N\psi=0, \tag{7}$$

where $v_g=k_0c^2/\omega_0$ is the group velocity of the light wave.

Next, we present the relevant equations for the EAWs in the presence of the light wave pressure. We have a Boltzmann electron density for the hot electrons,

$$N_h=\exp(\varphi-\Psi)\quad-\quad 1, \tag{8}$$

where $\varphi=e\phi/T_h$ and $\Psi=e^2|\psi^2|/2mc^2T_h$ are the normalized electron-acoustic wave and ponderomotive (due to the light wave pressure) potentials, respectively. The continuity and momentum equations for the cold electrons are

$$\frac{\partial N_c}{\partial t}+\frac{\partial}{\partial z}[(1+N_c)v_c]=0 \tag{9}$$

and

$$\left(\frac{\partial}{\partial t}+v_c\frac{\partial}{\partial z}\right)v_c=V_{Th}^2\frac{\partial(\varphi-\Psi)}{\partial z}, \tag{10}$$

where v_c is the fluid velocity of the cold electrons and $V_{Th}=(T_h/m)^{1/2}$ is the thermal speed of the hot electrons. Equations (8)–(10) are closed by means of Poisson's equation

$$\lambda_{Dh}^2\frac{\partial^2\varphi}{\partial z^2}=N_h+N_c\delta. \tag{11}$$

We consider two kinds of plasma slow responses associated with the electron-acoustic waves. First, we neglect the nonlinear terms in the plasma slow response and obtain from Eqs. (8)–(11) with $|\varphi-\Psi|\ll 1$,

$$\left(\frac{\partial^2}{\partial t^2}-C_e^2\frac{\partial^2}{\partial z^2}-\lambda_{Dh}^2\frac{\partial^4}{\partial t^2\partial z^2}\right)\varphi =\frac{e^2}{2mc^2T_h}\left(\frac{\partial^2}{\partial t^2}-C_e^2\frac{\partial^2}{\partial z^2}\right)|\psi|^2. \tag{12}$$

Equations (7), (11), and (12) are the desired equations [17] governing the dynamics of coupled light and linear EAWs. The present system of equations significantly differs from that of Karpman [1,2].

Second, we consider the dynamics of the nonlinear EAWs in the presence of light waves. Here, we write

$$N_h\approx\varphi-\Psi+\frac{1}{2}\varphi^2, \tag{13}$$

and retain the nonlinear flux N_cv_c and the convective nonlinearity $v_c\partial v_c/\partial z$. We then normalize the time and space variables by ω_{ph}^{-1} and λ_{Dh}, the fluid velocity v_c by V_{Th}, introduce the stretched variables $\xi=\epsilon^{1/2}(z-\lambda\tau)$ and $\tau_s=\epsilon^{3/2}\tau$, expand N_c, $V_c(=v_c/V_{Th})$, and φ in a power series of the smallness parameter ϵ, and take $e^2|\psi|^2/2m^2c^2V_{Th}^2\sim\epsilon^2$. Hence to lowest order, we have a set of equations that gives the constant $\lambda=\sqrt{\delta}$. To next order in ϵ, we have a set of equations that are combined to obtain

$$\frac{\partial\varphi}{\partial\tau_s}+\frac{\sqrt{\delta}}{2}\frac{\partial^3\varphi}{\partial\xi^3}-\frac{(3+\delta)}{2\sqrt{\delta}}\varphi\frac{\partial\varphi}{\partial\xi}=0, \tag{14}$$

which is a Kortweg–de Vries equation [18] without a light wave driver. It turns out that within the scheme of our time and space scales, the nonlinear EAWs are decoupled with the light waves. However, the nonlinear EAWs affect the light envelopes since in Eq. (7) the interaction potential is of the form $N=\partial^2\varphi/\partial\xi^2$.

The newly derived coupled equations in Sec. II may admit interesting envelope soliton solutions comprising density cavities trapping light waves. They also describe the collapse of light waves even in one space dimension. The collapse occurs when the light waves are modulated by nonresonant long-wavelength (in comparison with λ_{Dh} electron-acoustic perturbations with $\partial\varphi/\partial t\neq C_e\partial\varphi/\partial z$. Here, we have

$$\phi \sim \frac{e|\psi|^2}{2mc^2}, \tag{15}$$

so that Eq. (7) takes the form

$$2i\omega_0\left(\frac{\partial}{\partial \tau}+v_g\frac{\partial}{\partial z}\right)\psi + c^2\frac{\partial^2\psi}{\partial z^2} - \frac{\omega_{ph}^2}{8\pi n_{h0}mc^2}\psi\frac{\partial^2|\psi|^2}{\partial z^2}=0, \tag{16}$$

which contains a nonlocal nonlinearity. The latter is responsible for the light wave collapse, as discussed by Litvak and Sergeev [19]. Specifically, substituting $\psi=\psi(z-v_g\tau)\exp(i\eta\tau)$ into Eq. (16), one finds a cusp-shaped light wave soliton in the steady state. We have

$$|\psi|^2=\psi_0^2\mathrm{sech}^2\left[\left(\frac{2\eta\omega_0}{c^2}\right)^{1/2}(z-v_g\tau)+\left(1-\frac{|\psi|^2}{\psi_0^2}\right)^{1/2}\right], \tag{17}$$

where $\psi_0^2=m^2c^4/e^2$. The dynamics of the solution of Eq. (16) for an arbitrary field distribution can be studied by using the two integrals [19] of the nonlinear Schrödinger equation. The integrals show that due to a nonlocal nonlinearity, the evolution of any initial distribution leads to the appearance of a singularity associated with the sharpening of the vector potential profile.

On the other hand, one can also study the stationary solution of

$$c^2\frac{\partial^2\psi}{\partial\zeta^2}+2\omega_0\Omega\psi-\omega_{ph}^2\lambda_{Dh}^2\frac{\partial^2\varphi}{\partial\zeta^2}\psi=0 \tag{18}$$

by inserting the steady-state soliton solution of Eq. (14), viz. $\varphi=-\varphi_0\mathrm{sech}^2(\zeta/\lambda_s)$, where Ω is a nonlinear frequency shift, φ_0 is the minimum potential at $\zeta=|z-v_g\tau_s|=0$ and λ_s is the width of the soliton. It turns out that the light waves can be quantized [20] in a given nonlinear electron-acoustic wave potential.

III. SUMMARY

In summary, we have considered the nonlinear interactions of large amplitude light waves with the electron-acoustic perturbations in a two-electron temperature plasma. It is shown that the interaction gives rise to an envelope of light waves whose dynamics is governed by a nonlinear Schrödinger equation. The light wave envelope, in turn, affects (is affected by) the propagation of the linear (nonlinear) EAWs that are described by Eqs. (12) and (14). The newly derived system of equations is suitable for studying the formation of light envelope solitons, trapping of light waves in density cavities, and the collapse of light waves even in one dimension. The results of the present investigation are thus useful for understanding the nonlinear propagation of light waves in two-electron plasmas, such as those in space and laser-produced plasmas.

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